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A FUZZY APPROACH TO CRITICAL PATH METHOD IN PROJECTS PLANNING

BY

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Abstract. This paper presents a method to finding out critical path in the fuzzy project network. Activity times in the project network are represented by trapezoidal fuzzy numbers. We used a method based on ranking value of a fuzzy number. A numerical example is given. The results obtained by using the present method are compared with those obtained using defuzzification method used in literature.

Keywords: fuzzy project network; trapezoidal fuzzy numbers; ranking method; fuzzy total slack; defuzzification.

1. Introduction

A determined duration and cost for each activity, including normal durations and normal costs, are required for a successful implementation of critical path method.

In practice, sometimes, this requirement is difficult to fulfil since activity durations represent estimates of the actual time needed. Moreover, a significant amount of uncertainty there might be associated with the actual durations.

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During the usual business, but mostly when an action is being executed for the first time, the project manager finds it difficult to set a deterministic value for the time to execute the activity. In such situations, due to the lack of duration information or poor definitions of the activity, the fuzzy sets or fuzzy numbers can be used to model the activities and their durations (Elizabeth and Sujatha, 2013).

Ranking methods are very important in fuzzy critical path method (CPM). Manny methods of ranking fuzzy number have been proposed in literature: (Bortalan and Degani, 1985; Chen, 1985; Kim and Park, 1990). For ease of implementation, a useful ranking method (Liang and Han, 2004) is utilized. It is presented (Shankar *et al*., 2010) a defuzzification formula for trapezoidal fuzzy numbers.

This paper, based on a method of ranking value of a fuzzy number, solves out a fuzzy critical path problem with trapezoidal representation of activity duration. The results are compared with those obtained using defuzzification formula used in literature.

The ambiguities involved in the assessment activity times in a project network could be effectively improved by using this method, hence the management can get a more convincing and effective project management decision – making.

2. Fuzzy Concept

Let's consider the fuzzy number $A = (a, b, c, d)$ shown in Fig. 1 to be trapezoidal fuzzy number if its membership function was given by:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ 1 \quad\ , b \le x \le c \\ \frac{d-x}{d-c}, c \le x \le d \\ 0 \quad\ , otherwise \end{cases} \tag{1}$$

Let $A_i = (a_i, b_i, c_i, d_i),\ i = \overline{1,n}$ be fuzzy numbers with membership functions μ_{Ai} respectively.

Let $x_1 = Infimum\ D, x_2 = Supremum\ D$ and $D_i = \{x/\mu_{Ai}(x) > 0, i=1,n$.

Then, the ranking value of fuzzy number A_i is defined as:

$$R(A_i) = \beta \left[\frac{d_i - x_1}{x_2 - x_1 - c_i + d_i}\right] + (1-\beta)\left[1 - \frac{x_2 - a_i}{x_2 - x_1 + b_i - a_i}\right] \tag{2}$$

where the decision maker's risk attitude index β can be obtained by:

$$\beta = \left[\sum_{\substack{i \\ A_{ij} \in ACT}} \sum_j \frac{b_{ij} - a_{ij}}{(b_{ij} - a_{ij}) + (d_{ij} - c_{ij})} \right] / t \qquad (3)$$

The ACT represents the set of all activities and t represents the number of activities in the project network.

For a fuzzy number $A_i, i = \overline{1,n}$ with membership functions μ_{Ai} we define:

$$m_i = min\{x/\mu_{Ai}(x) = 1\} + max\{x/\mu_{Ai}(x) = 1\} \qquad (4)$$

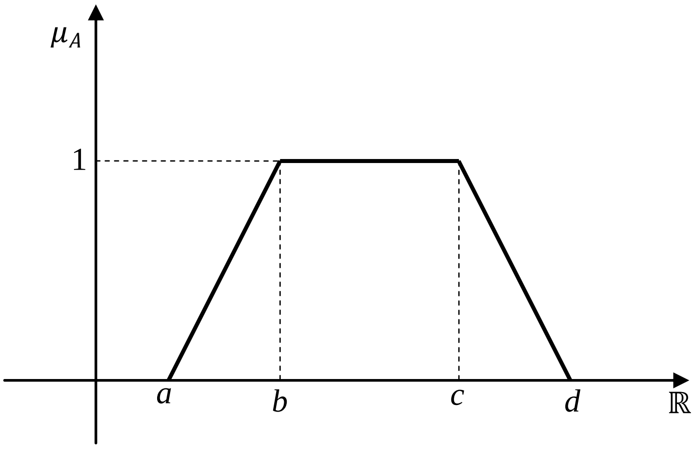

Fig. 1 – A trapezoidal fuzzy number.

Based on the following rules, (Han *et al*., 2006), the fuzzy numbers A_i and A_j are ranked:

$$A_i > A_j \Leftrightarrow R(A_i) > R(A_j), or\ R(A_i) = R(A_j)\ and\ m_i > m_j \qquad (5)$$

$$A_i = A_j \Leftrightarrow R(A_i) = R(A_j)\ and\ m_i = m_j \qquad (6)$$

Within a project network the fuzzy activity time, marked by FET_{ij}, of activity A_{ij} is represented by trapezoidal fuzzy number $FET_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ where a_{ij}, d_{ij} are minimum and maximum values of assessing activity time for A_{ij}. If there is no more information regarding the activity A_{ij}, fuzzy activity time $FET_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ can be evaluated subjectively by the decision-maker based on his knowledge, experience and subjective judgement.

The extended algebraic operations of any two fuzzy activity times $FET_1 = (a_1, b_1, c_1, d_1)$ and $FET_2 = (a_2, b_2, c_2, d_2)$ can be expressed as:

Addition $\oplus$:

$$FET_1 \oplus FET_2 = (a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) = \\ = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \quad (7)$$

Subtraction $\ominus$

$$FET_1 \ominus FET_2 = (a_1, b_1, c_1, d_1) \ominus (a_2, b_2, c_2, d_2) = \\ = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) \quad (8)$$

Let $FET_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ be the fuzzy activity time of activity A_{ij}.

Defuzification of fuzzy set is used to obtained crisp values which represent uncertain data. The decision-making process is improved when a fuzzy number is converted into a crisp number because it is easier to make comparison between two or more fuzzy numbers. Many defuzification methods are in use for different types of fuzzy numbers (Oladeinde and Itsisor, 2013; Oladeinde and Oladeinde, 2014). There we can get the crisp value representing the fuzzy number using the expression (Shankar *et al*., 2010) for a trapezoidal fuzzy number $A = (a, b, c, d)$:

$$Centroid(A) = \frac{(c^2+d^2+cd)-(a^2+b^2+ab)}{3[(c+d)-(b+a)]} \quad (9)$$

This formula can be used to defuzzify the fuzzy total slack of the activity. A critical activity can be defined as the one whose total slack is equal to zero.

3. Computing Fuzzy Time Values in a Fuzzy Project Network

A fuzzy project network is an acyclic digraph. The vertices show events whilst the direct edges show the activities to be performed in the project.

Let's consider $V = \{v_1, v_2, \dots, v_n\}$ to be a set of vertices, where v_1 and v_n represent the starting and ending events of the project and each v_i belongs to some path from v_1 to v_n. The activity $A_{ij} = (v_i, v_j)$ shows the activities to be performed in the project. Activity A_{ij} is represented by one and only one arrow between events v_i and v_j, where $i < j$.

A fuzzy FET_{ij} number is defined for each activity A_{ij}, where FET_{ij} is the fuzzy time needed for the completion of A_{ij} (Khalaf, 2013). The longest path from v_1 to v_n is the critical path. An activity A_{ij} on the critical path is called a critical activity.

Let's FES_i and FLS_i be the fuzzy earliest start of event i and the fuzzy latest start of event i, respectively. Let FEF_j and FLF_j be the fuzzy earliest finish of event j and the fuzzy latest finish of event j, respectively (Liang and Han, 2004).

According to CPM, the forward pass yields the fuzzy earliest start and fuzzy earliest finish times:

$$FES_j = \max_i\left\{FES_i \oplus FET_{ij} / i \in NP(j), j \neq 1, j \in \mathbb{N}\right\} \quad (10)$$

$$FEF_j = FES_i \oplus FET_{ij} \quad (11)$$

To identify the maximum value where FES_j is the fuzzy earliest – beginning time with $FES_1 = (0,0,0)$ at the initial node, ranking value is used. $NP(j)$ represents the set of all nodes connected to all predecessor activities of node j, *i.e.* $NP(j) = \left\{i/A_{ij} \in \mathbb{N}\right\}$, whilst $\mathbb{N}$ represents the set of all nodes in a project network.

The backward pass is performed to calculate the fuzzy latest beginning and fuzzy latest end times:

$$FLS_i = FLF_j \ominus FET_{ij} \quad (12)$$

$$FLF_j = \min_k\left\{FLF_k \ominus FET_{jk} / k \in NS(j), j \neq n, j \in \mathbb{N}\right\}, j = \overline{n-1,1} \quad (13)$$

To identify the minimum value where $FLF_n = FES_n$, ranking value is used. $NS(j)$ represents the set of all nodes connected to all successor activities of node j, *i.e.* $NS(j) = \left\{k/A_{jk} \in S(j), j \in \mathbb{N}\right\}$ whilst $S(j)$ is the set of all successor activities of node j.

Once $FES_i, FEF_j, FLS_i, FLF_j$ have been determined for the A_{ij} activity, the total slack fuzzy time is calculated using the equation:

$$FTS_{ij} = FLF_j \ominus \left(FES_i \oplus FET_{ij}\right), 1 \leq i < j \leq n, i, j \in \mathbb{N} \quad (14)$$

4. Algorithm to Find Fuzzy Critical Path in a Fuzzy Project Network

Let's consider a fuzzy project network where the duration of each activity in the fuzzy project network is shown by trapezoidal fuzzy member.

The algorithm used to determine a fuzzy critical path and the fuzzy slack is shown in (Deepti, 2010):

1) Define the fuzzy project network and all its relevant activities or tasks;

2) Set the relationship among the activities. Select which activities are to precede and which are to follow others;

3) Draw the fuzzy project network connecting all the activities;

4) Assign fuzzy time estimates to each activity;

5) Calculate the fuzzy longest time path through the network by using appropriate ranking formula to:

a) Let $FES_1 = (0,0,0)$ and calculate $FES_j, j = \overline{2,n}$ using Eq. (10);

b) Let $FLF_n = FES_n$ and calculate $FLF_j, j = n-1, n-2, \ldots, 1$ using Eq. (13);

6) Calculate the fuzzy total slack FTS_{ij} using Eq. (14);

7) Find out all the possible paths P_k and calculate the total slack fuzzy time of path $P_k, FCPM(P_k)$ in a project networkby:

$$FCPM(P_C) = \sum_{\substack{1 \le i < j \le n \\ i,j \in P_K}} FTS_{ij}, P_K \in P \qquad (15)$$

8) Determine the fuzzy critical path by using the following:

$$FCPM(P_C) = \min \ \{FCPM(P_k)/P_k \in P\} \qquad (16)$$

9) Find out the level of membership so that the project can be completed in due time.

5. Numerical Example

Let's consider a hypothetical numerical example in order to show the method described before.

To generate a new kind of goods, a company has to run a relevant project that contains six different activities presented in Fig. 2. The fuzzy activity time for each activity is presented in Table 1. We target to identify the fuzzy critical path.

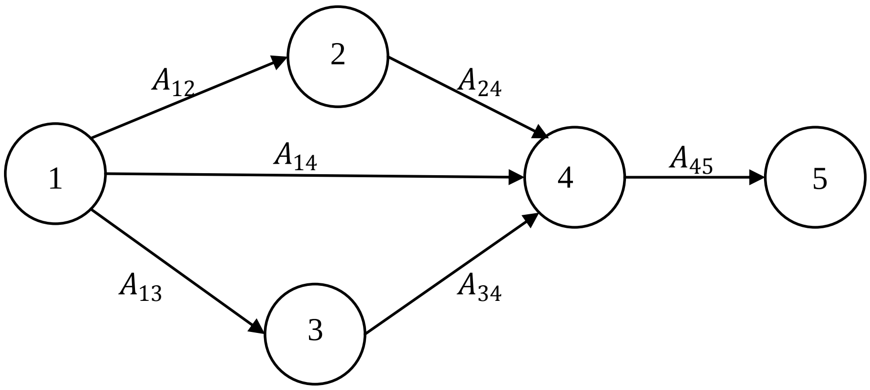

Fig. 2 – A project network.

Table 1
Fuzzy Activity Time for Each Activity

Nr.	Activity Code	Activities directly preceding	Fuzzy activity time (weeks)
1	A_{12}	-	$(3,5,5,7)$ *approximately 5 weeks*
2	A_{13}	-	$(5,10,10,15)$ *approximately 10 weeks*
3	A_{14}	-	$(1,3,4,5)$ *approximately between 3 and 4 weeks*
4	A_{24}	A_{12}	$(2,4,5,6)$ *Approximately between 4 and 5 weeks*
5	A_{34}	A_{13}	$(1,2,3,4)$ *approximately between 2 and 3 weeks*
6	A_{45}	A_{14}, A_{24}, A_{34}	$(6,8,10,11)$ *approximately between 8 and 10 weeks*

To implement the methodology presented above a MATLAB computer program is developed. It will run project network fuzzy arithmetic calculations.

The conclusions of the fuzzy critical path analysis are presented in Table 2 below together with the fuzzy total slack.

Table 2
Values of FET_{ij}, FES_{ij}, FEF_{ij}, FLS_{ij} and FLF_{ij} and Fuzzy Total Slack of Each Activity

Act. Code	Duration (weeks)	Fuzzy Earliest Start	Fuzzy Earliest Finish	Fuzzy Latest Start	Fuzzy Latest Finish	Fuzzy total slack
A_{12}	$(3,5,5,7)$	$(0,0,0,0)$	$(3,5,5,7)$	$(-12,0,6,19)$	$(-5,5,11,22)$	$(-12,0,6,19)$
A_{13}	$(5,10,10,15)$	$(0,0,0,0)$	$(5,10,10,15)$	$(-18,-3,3,18)$	$(-3,7,13,23)$	$(-18,-3,3,18)$
A_{14}	$(1,3,4,5)$	$(0,0,0,0)$	$(1,3,4,5)$	$(-4,6,12,23)$	$(1,10,15,24)$	$(-4,6,12,23)$
A_{24}	$(2,4,5,6)$	$(3,5,5,7)$	$(5,9,10,13)$	$(-5,5,11,22)$	$(1,10,15,24)$	$(-12,0,6,19)$
A_{34}	$(1,2,3,4)$	$(5,10,10,15)$	$(6,12,13,19)$	$(-3,7,13,23)$	$(1,10,15,24)$	$(-18,-3,3,18)$
A_{45}	$(6,8,10,11)$	$(6,12,13,19)$	$(12,20,23,30)$	$(1,10,15,24)$	$(12,20,23,30)$	$(-18,-3,3,18)$

All possible paths are $P = \{(1,2,4,5), (1,4,5), (1,3,4,5)\}$. For all of them we calculate the fuzzy completion time and the ranking value of fuzzy completion time. Obtained values are presented in Table 3.

Table 3
Fuzzy Activity Time for Each Activity

Nr.	Paths	Fuzzy completion time of path P_i $FCPM(P_i)$	Ranking Value of $FCPM(P_i)$
1	$P_1 = (1,2,4,5)$	$FCPM(P_1) = (-42,-3,15,56)$	$R\big(FCPM(P_1)\big) = 0{,}5676$
2	$P_2 = (1,4,5)$	$FCPM(P_2) = (-22,3,15,41)$	$R\big(FCPM(P_2)\big) = 0{,}5834$
3	$P_3 = (1,3,4,5)$	$FCPM(P_3) = (-54,-9,9,54)$	$R\big(FCPM(P_3)\big) = 0{,}5274$

Because $R(FCPM(P_3)) < R(FCPM(P_1)) < R(FCPM(P_2))$ the fuzzy critical path is P_3. Hence, the project completion time is takes between 20 and 23 weeks, *i.e.* $(12,20,23,30)$. The level of membership that the project's completion time being within 18 weeks is 0.75.

Next step it is to defuzzify the total float of each activity by using formula (9). Then we get to the results present in the Table 4.

Table 4

Defuzzified Values of Total Slack Time for Each Activity

Act. Code	Duration (weeks)	Fuzzy total slack	Defuzzified value
A_{12}	$(3,5,5,7)$	$(-12,0,6,19)$	3.3063
A_{13}	$(5,10,10,15)$	$(-18,-3,3,18)$	0
A_{14}	$(1,3,4,5)$	$(-4,6,12,23)$	9.3030
A_{24}	$(2,4,5,6)$	$(-12,0,6,19)$	3.3063
A_{34}	$(1,2,3,4)$	$(-18,-3,3,18)$	0
A_{45}	$(6,8,10,11)$	$(-18,-3,3,18)$	0

Analysing the values from Table 4 we can conclude that the activities A_{13}, A_{34} , A_{45} are critical because their crisp slacks are equal to 0 and the critical path is $P_3 = (1,3,4,5)$. This result is consistent with the solution we got by the method based on ranking value of a fuzzy number.

3. Conclusions

In order to find out the critical path we have computed fuzzy completion time for each path in a project network. A numerical example was resolved using the method based on ranking value of trapezoidal fuzzy numbers. The result was then compared with the result obtained by using defuzzification formula for trapezoidal fuzzy number.

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O ABORDARE FUZZY A METODEI DRUMULUI CRITIC ÎN PLANIFICAREA PROIECTELOR

(Rezumat)

Această lucrare prezintă o metodă de a găsi drumul critic într-un proiect în care duratele activităților sunt exprimate prin numere fuzzy. În reprezentarea duratelor activităților din proiect sunt utilizate numere fuzzy trapezoidale. S-a folosit o metodă de ordonare a numerelor fuzzy. Rezultatele obținute utilizând metoda prezentată sunt comparate cu rezultatele obținute printr-o metodă de defuzzificare utilizată în literatura de specialitate.